#### RESEARCH



# **Overt Visual Attention in the Formation of Preference Between Complex Lottery Options**

Xinhao Fan<sup>1,2</sup> · Jacob Elsey<sup>1</sup> · Aurelien Wyngaard<sup>1,3</sup> · Aaron L. Sampson<sup>1</sup> · You-Ping Yang<sup>1,8</sup> · Erik E. Emeric<sup>1</sup> · Moshe Glickman<sup>5,6</sup> · Marius Usher<sup>4</sup> · Dino Levy<sup>7</sup> · Veit Stuphorn<sup>1</sup> · Ernst Niebur<sup>1</sup>

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#### Abstract

Our ultimate goal is to understand mechanisms of decision-making, a fundamental cognitive function. Models of multiattribute decision-making vary on whether preference formation is based on within-option or within-attribute processing. We carry out a combined empirical and computational study using lottery options with varying task complexities. We monitor eye gaze during the decision formation to determine which decision-relevant information participants attend and when. We compare models of different levels of complexity in their ability to account for the choices made by individual participants. We find that two models outperform all others. The first is the two-layer leaky-competing accumulator based on prospect theory (LCA-PT), which predicts human choices on simple tasks better than any other model. For complex tasks a new model based on operations research performs best, with both its performance as well as that of the second-ranked LCA-PT model significantly exceeding that of all other models. Both models use the sequence of observed eye movements for each participant to capture the allocation of attention to specific options and attributes during the decision process, but make different assumptions about the effect of attention on decision-making. Our results suggest that, when faced with complex choice problems, people form preferences primarily based on attention-guided pairwise, within-attribute value comparisons. Suboptimal decisions are at the basis of many societal ills, from drug abuse to eating disorders to displaying inappropriately violent behavior. Understanding their underlying mechanisms has the potential of developing remedies for these maladaptive behaviors.

Keywords Decision making  $\cdot$  Multi-attribute decision-making  $\cdot$  Selective attention  $\cdot$  Preference formation  $\cdot$  Computational modeling

# Introduction

Making decision between complex choice options (which are characterized by multiple attributes or dimensions) is one of the most difficult tasks that we frequently encounter in our daily life. This is often due to the problem of integrating multiple pieces of information which present us with various trade-offs—one option may be better with regards to one dimension (or attribute), e.g., quality of a product, while another with regards to another attribute, e.g., its cost. Risky choice is a particular type of multi-attribute choice which has been subject to extensive research. It presents participants with choices between lottery options and is the focus of our study. A number of normative theories were developed to prescribe good choices in multi-attribute decisions (Keeney risky choice, in particular, the normative theory is *Subjective Expected Utility* (Savage, 1954). These normative theories, and even more recent descriptive theories of risky choice, such as Prospect Theory (Kahneman & Tversky, 1979; Tversky & Kahneman, 1992), compute an overall value for each option in the form of a weighted average, with high values in one attribute compensating for low values in others.<sup>1</sup> Empirical research in decision-making, however, indicates marked violations of the normative theory. Those violations have indicated a variety of context effects (Huber et al., 1982; Simonson, 1989; Busemeyer et al., 2019; Berkowitsch et al.,

et al., 1993), which are often described as *compensatory*. For

Extended author information available on the last page of the article

<sup>&</sup>lt;sup>1</sup> The choice is assumed to stochastically select the option with the highest subjective expected value. In the Prospect Theory, this value is being estimated using subjective values and subjective probabilities; however, the theory preserves the idea that value is integrated across all possible outcomes.

2014) in which the relative preference between two choice options depends on options which are *not* chosen, violating basic principles of choice rationality (Von Neumann & Morgenstern, 1947).

To deal with these violations, and also to face the challenge that the normative theory requires processing resources that exceed the working memory capacities of humans (under normal, unaided, conditions), a set of heuristic models have been proposed in which the formation of preferences does not involve the computation of an overall value for each option (Tversky, 1969, 1972; Gigerenzer & Goldstein, 1996). One important distinction between multi-attribute decisionmaking models is whether processing occurs mainly withinoption or within-attribute (Fellows, 2006; Payne et al., 1993). While the normative theory which estimates an overall value for each option requires within-option processing, heuristic models involve within-attribute processing. As shown by Fellows (2006), lesions in the ventromedial prefrontal cortex can result in changes between these types of decision strategies.

From the information processing point of view, an advantage of within-attribute processing is that choice options are compared along a single attribute at a time, rather than having to integrate them into options (within-options processing). Alternatives are chosen or eliminated based on such comparisons (Tversky, 1969, 1972; Gigerenzer & Goldstein, 1996). This saves on processing resources as decisions can proceed based on partial information. Research in which task complexity was manipulated has shown that with increasing task complexity, participants are more likely to rely on non-compensatory, within-attribute decision strategies (Payne et al., 1993). Much of this research has relied on a (computer-) mouse-driven attribute sampling paradigm, commonly referred to as *Mouselab*, a term we also adopt due to its wide use. In this paradigm, choice options are presented on a hidden matrix which requires the participants to reveal information by sequential clicks of a computer mouse (Johnson et al., 1989). More recently, Glöckner and Betsch (2008) have argued that the Mouselab design/method induces an unnatural environment that increases the reliance on heuristic (non-compensatory) strategies. Lohse and Johnson (1996) compared behavior in the Mouselab environment with behavior based on eye tracking and reported that participants using the former tend to have more systematic information acquisition behavior than in the latter. This may be due to the requirement for deliberate, more timeconsuming and sequential actions (mouse movements and clicks) in *Mouselab* than is necessary while making natural eye movements. Using free viewing, Glöckner and Betsch (2008) have shown that most subjects deploy compensatory rapid strategies in probabilistic inferences task (which is a type of multi-attribute decision with binary cues). Brusovansky et al. (2018) have reached the same conclusion in classical multi-attribute decision tasks. We also point out that in natural free viewing (without the masking protocol we employ in our study, see below), there may be some degree of parallel processing mediated by the information gathered in peripheral vision. To be clear, in our paradigm, all peripheral information is available, with the only exception being that which is covered by the masks obscuring the values of nonfixated attributes. Future studies may be needed to examine if the effects we obtain here generalize to free viewing.

Several models of decision-making have an attentional component in which options are selected, e.g., Busemeyer and Townsend (1993); Roe et al. (2001); Johnson and Busemeyer (2010); Birnbaum (2008). However, the attentional state of the decider is typically considered as a hidden (internal) stochastic process, in which there are random jumps between possible attentional states. Using sophisticated computational models, Trueblood et al. (2022) showed in a recent study that context effects including preference reversals (and their absence) can be explained by stochastic changes of attention.

It is possible, however, to obtain access to this internal process by monitoring eye gaze, i.e., overt attention. While a large literature exists about measuring the effects of covert attention (review: Carrasco (2011)), a simpler procedure is to replace covert attention by overt attention. It has been known for more than a century that covert attention can be dissociated from overt attention (von Helmholtz, 1896), but the latter is often a good approximation of the former. In the realm of visual perception, it has been shown that predictions of computational models of covert attention (Niebur & Koch, 1996; Itti et al., 1998) correlate strongly with eye movements (Parkhurst et al., 2002).

Following previous work in the decision-making literature (Russo, 1978; Russo & Dosher, 1983; Glöckner & Herbold, 2011; Stewart et al., 2016; Krajbich et al., 2010; Glickman et al., 2019), we also measure eye movements to determine the attention state, as described below.

In a recent study from our labs (Glickman et al., 2019), we have examined the behavioral mechanisms that humans use by monitoring their gaze while they made choices between simple lotteries of the form (win amount x with probability p). We then examined a number of within-option and within-attribute choice models, which were constrained by the observed scan path (gaze location over time) while participants collected information about their choices. Consistent with results by Glöckner and Betsch (2008, 2012) and Brusovansky et al. (2018), we found a predominance of within-option processing, subject to some inter-individual variability. Since the choice task, however, only involved the simplest choice options (two options with two attributes each), it is possible that this conclusion is due to the low task complexity. To examine the dependency of the decision mechanism (in particular, the use of within-option vs. withinattribute processing) on task complexity, in this study, we carried out a novel experiment in which we examine multiattribute decisions while the task complexity is manipulated in a within-participant design. While the options in the previously used simple task were defined by a total of four pieces of information (two options with two attributes each), this number is substantially increased to 16 in the complex task (four options with four attributes each). We then implement 15 computational models designed to predict choices made by participants by optimizing model parameters based on behavioral variables of each participant, including their eye movements. The relative performance of models is compared using several metrics in order to infer the impact of task complexity on the decision and attentional process.

#### **Aims of This Study**

The aims of this work are to study the influence of attention on choice in multi-attribute decision-making. Given limitations of cognitive processing, the role of attention may increase with complexity of the decision space which motivates us to vary task complexity. Our approach is to combine an empirical study of behavioral choice by healthy human volunteers with computational modeling. Both methods are very powerful, and importantly, they complement each other: The empirical results constrain the models, and the models provide quantitative predictions of choice behavior and an understanding of the underlying mechanisms. Using the models, we aim to better understand the type of process (within-attribute/alternative) that takes place at each level of complexity and to understand the impact of the eye gaze information in accounting for the choice data. Moreover, using the best fit model parameters, we can understand the algorithm that is being deployed (e.g., the role of memory).

There are two main parts to this combined empirical and computational study:

(i) Collecting empirical results from a new experimental paradigm. It varies from most previous work in two ways: (1) we study situations of varying complexity to address the question of whether potential shifts in choice strategy (between within-attribute and within-option searches) depend on complexity, and (2) we monitor the state of overt attention (scanpaths) during the decision process. This allows us to constrain models with this information and determine whether they can explain behavior more successfully than models that are not privy to this information.

(ii) The effect of several variables on choice behavior in situations of different complexity is quantitatively assessed in a substantial number (N=15) of computational models. The main emphasis is to study the effect of selective attention on choice in these models which are selected from three families: (1) descriptive models (i.e., not process models), (2) process models without attention, and (3) process models with attention. We study the impact of inter-trial interactions

on choice in the latent variable model which is part of the second family.

To anticipate our results, we find that while for simple choices the within-option models dominate, the situation reverses at high complexity choices.

#### Methods

#### **Experimental Methods**

We study the behavior of human participants in a multiattribute decision task. Each option is defined by multiple attributes. We investigate two cases that differ substantially in complexity. In the simpler case  $(2 \times 2)$ , two options are presented with two attributes each, while in the more complex situation  $(4 \times 4)$ , there are four options with four attributes each. Obviously, for an exhaustive evaluation of all aspects, a minimum of four attributes needs to be assessed in the first task, while the more complex one requires 16. Options are presented on a computer screen, and importantly, the values of all attributes are by default covered by opaque circles whose colors indicate the type of attribute but not its value. The value is "hidden" under the opaque disks and only shown when the participant actively fixates it. This is accomplished by the use of an eye tracker that continuously monitors the eye position of the head-fixed observer and permits attribute value unmasking within a few tens of milliseconds (frame rate 60/s), making the switch barely noticeable. Disk colors were consistent for all experiments: yellow for amount to win, blue for probability to win, red for amount to lose, and green for the delay until feedback becomes available, see Fig. 1A. The two latter attributes were only used for the  $4 \times 4$ task.

The options for each choice pairing were determined by systematically sampling one of five magnitudes for each attribute type. "Dominated" trials were cases in which all attributes of one option were superior to those belonging to other options  $(2 \times 2 \text{ task: } 60\% \text{ of trials}, 4 \times 4 \text{ task: } 20\% \text{ of}$ trials). While performance on dominated trials allows us to confirm that participants understand the task and to assess their state of vigilance, our main interest is to understand the choices humans make in non-trivial trials, in which each option is better in one attribute than all other options. A compromise between attributes is then required that depends on the individual preferences of the participants. In such "nondominated" trials ( $2 \times 2$  task: 40% of trials,  $4 \times 4$  task: 80% of trials), each option can be characterized by the attribute type in which it is best. Thus, in the  $2 \times 2$  task, we can distinguish between options with the higher amount to win ("Win+" option) and options with the higher probability to win ("Prob+" option). In the  $4 \times 4$  task, in addition to these option types, there is an option with the lowest for amount to



**Fig. 1** Task design. **A** Choice menu layout and attribute types. Left: Examples of two-option, two-attribute  $(2 \times 2)$  configurations. Right: four-option, four-attribute  $(4 \times 4)$  configurations. **B** Task flow for one  $2 \times 2$  trial. After fixating the center of the screen, participants are presented with the stimulus array. Symbols are masked by colored circles, see text. A mask is removed, and the underlying symbol revealed when the participant fixates it (dashed circles; dashed lines are saccades). Participants gather information about the options until they choose one of them by a button press. After the choice is made, the chosen option is displayed for 1 s, with all its attributes visible. Then, the amount of the gain or loss, if any, is displayed, and the next trial starts. **C** Attribute fixation types. Left, within-option: Saccades (orange arrows) from one

attribute type to another within the same option. Right, within-attribute: Saccades (purple arrows) within the same attribute type across different options. **D** Example attribute sampling strategies. Overlaid black lines are eye tracks of one participant each in one trial. Top: The filtering sampling strategy began with within-attribute saccades to the participant's attribute of preferred interest (probability to win), followed by within-option saccades in the option containing the superior magnitude of this attribute. Bottom: The exhaustive sampling strategy was comprised primarily of sequential within-option saccades to each option. Fixation patterns were typically spatial, i.e., inspecting options left to right or top to bottom

lose ("Loss+" option) and an option with the shortest delay until feedback ("Delay+" option). In the  $4 \times 4$  task, we used a Latin square design to ensure each option contained one attribute that was 1st, one that was 2nd, 3rd, and 4th ranked (Fig. 2B). Four of the five possible attribute magnitudes were used on a given trial (Fig. 2C). Probability magnitudes ranged between 10 and 90%; therefore, no option resulted in a certain outcome. Figure 2A shows examples of dominated and non-dominated attribute pairings in the  $2 \times 2$  task.

Figure 1A shows the stimulus configurations used in the experiment. The task was a free-viewing paradigm. Participants could collect all information they desired by looking selectively at those attributes they were interested in at a given point in time, as many times as they desired, until they indicated the choice of their preferred option by pressing the associated key on a keyboard. Figure 1B shows an illustra-

tion of all phases of one trial, starting when the participant directed their gaze at a fixation spot, then at a series of locations representing the 2 attributes of the available 2 options in this particular trial, and terminating with the selection of one of the options by pressing a key.

In both the  $2 \times 2$  and  $4 \times 4$  experiment, participants were informed that the outcomes of 10% of randomly selected trials would be paid out in real money to incentivize them to maximize their earnings. This conversion of nominal rewards to actual monetary rewards happened after all trials were completed. While performing the task, participants were not informed whether a given outcome was included in the pay out or not. Payment was in a lump sum at the end of the experiment, without providing information about which outcomes contributed to the computation of that sum. If earnings were negative after all trials were completed in the  $4 \times 4$  experiFig. 2 Stimulus design. A In the  $2 \times 2$  experiment, options were drawn from a 5x5 dimensional grid. Red: Example dominated choice pairing. Blue: Example of non-dominated choice pairing. B Example of non-dominated choice menu in the  $4 \times 4$  experiment. Each light-gray bar represents one option and the colored circles its attributes. A Latin square design was used to ensure that each option was uniquely best, second-best, third-best, and fourth-best in a particular attribute type; ranks are indicated by the colors (see text) in the circles of each attribute. C All attribute magnitudes used in the  $4 \times 4$  experiment



ment, i.e., if a participant made a net loss, they were adjusted to zero. For details on experimental methods, see Supplemental Section S1.

#### **Computational Methods**

Our goal is to understand the decision mechanism in multiattribute decision tasks of varying complexity by developing quantitative models of the preference formation process. We consider 15 specific models, subdivided into three families, that embody a range of different functional mechanisms underlying decision-making. For more details about all models, see Supplementary Section S3. Detailed mathematical equations are provided in Supplementary Section S4. Models are fitted to the data of each participant to find the model parameters that provide the best account for their choices. Accordingly, all model parameters are determined separately for each participant by maximizing the likelihood that the model predicts this participant's choices made in the two experimental tasks described earlier.

#### Models Based on Choice Information Only

The first family of models we consider, described in Section S3.1, consists of models whose behavior is determined entirely by the choice values describing the options available in each trial. For these models, it is assumed that the specific combination of attributes of each option can be directly transformed into a value estimate for this option. The simplest model is maximizing the expected value (EV) of collected rewards (amounts). Not all humans can, or want to, maximize this quantity. Instead, they may maximize a subjective value (SV) which is specific to each observer. A simple way to compute such a value is the sum of weighted attribute values

for a given option; we call this the additive rule (AR). Alternatively, an SV can be computed as a nonlinear combination of attribute values. This approach is taken in prospect theory (PT). In this report, SV always refers to the subjective value computed from prospect theory, as defined in Section S3.1.

Models in this family assume that all available information about the attributes is used for decision-making, and these models do not take into account the dynamic processes during the decision process. In contrast, in process models, discussed in the following sections, during each trial, some functional process transforms information about attributes into choice preferences for the available options. Assumptions about the nature of the underlying process distinguish the models.

#### **Process Models Without Attention**

The three models in this family are described in Section S3.2. In the latent variable (LV) model, we investigate the influence of inter-trial interactions on choice. That is, we consider that behavior may be influenced by the responses in previous trials. This is represented in the formal model by a number of internal latent variables related to the reward history that contribute to the computation of the subjective value, resulting in dynamic, history-dependent changes in behavior.

The other two models are related to two attention-based models that we summarize in the following "Process Models with Attention" section. These two models do not compute an explicit value function for individual options; instead, they rank the options by comparing their attributes. The first is the decision by sampling (DbS) model (Stewart, 2009) in which the decision is based on a series of binary comparisons between attributes. Eventually, the option is chosen that has the highest number of favorable comparisons. The second model is the Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEE) (Brans & Vincke, 1985), an algorithm developed in Management Science/Operations Research. It likewise compiles results of comparisons of pairwise attribute values in a systematic way to arrive at a choice between options.

#### **Process Models with Attention**

Finally, in Section S3.3, we introduce models in which attention has a specific functional role in decision-making. These models explicitly take into account (overt) attentional influences, i.e., eye movement data. First, in two versions of a Bayesian approach, we assume that participants perform Bayesian inference based on the sequence of fixations to optimize either EV (the BI-EV model) or SV (the BI-SV model). In another approach, the attention-modulated driftdiffusion (aDDM) (Krajbich & Rangel, 2011) model uses evidence accumulation at each fixation. In the Leaky Competing Accumulator (LCA) model (Usher & McClelland, 2001, 2004), integration of value occurs by attentional selection of attributes in a lossy process. The algorithm then assigns values to each available option and finally selects the highest value. Following Glickman et al. (2019), we assign option values using prospect theory, which includes simpler value-assignment methods, like EV and EU, as special cases. We call this model LCA-PT.

As indicated earlier, we extended three existing models. For each of them, the major modification is to replace the hidden stochastic attentional process with the observed one, something that is possible in our experimental paradigm but difficult or impossible in most earlier work. For the first of these models, decision by sampling, we assumed that the binary comparisons that in the original model (Stewart, 2009) occur randomly (or exhaustively, as in our implementation of that model, see \$3.2.2) are in fact controlled by attention, resulting in the attention-modulated DbS model or aDbS model. The second model is decision field theory, which assumes in its original form (Roe et al., 2001) that attention switches stochastically between attributes. We introduce two modified versions in which stochastic switching of attention is replaced by the observed changes of attention for each participant to either option values (attention-controlled decision field, the aDFT model) or attribute values (attribute-specific attentional decision field theory, the aaDFT model) in each trial. The third modified model is PROMETHEE, where in our version we take into account the sequence of information selection determined by eye movements. In addition, we also add memory leaks to arrive at the attention and memorymodulated PROMETHEE model or AMP model.

# **Model Recovery and Parameter Recovery**

We tested if the models we consider can be recovered from choice and gaze data of the type produced in our experiment. We generated synthetic data for each of the models we are considering for a population of "simulated participants" (SPs), see Supplementary Sections S8 and S9 on procedural details. SPs were presented with the choice problems presented to our participants and with their gaze patterns. The model generates a response to these inputs, for each choice trial, that corresponded to our synthetic data. We then fitted the synthetic data with all the choice models and selected the best one based on their AIC scores<sup>2</sup>

The recovery matrices in Fig. 3 show on the ordinate the fraction of synthetic data generated by Model y that is recovered as Model x on the abscissae. Diagonal terms correspond to correct recoveries. Please see Supplementary Section S9 for details.

As we see in Fig. 3, most models show a reasonable recovery. One exception is the PROMETHEE model which is confused with the DbS model in both tasks and, in addition, with the AR model in the  $4 \times 4$  task.

In addition, we carried out a parameter recovery exercise. For this aim, we generate new sets of synthetic data, but for each SP, we only fitted with the generating model. We then plotted the correlation between the generating and the recovered parameters. The results shown in Table 3 in the supplementary material indicate reasonable parameter recovery. See Section S8 for details.

# Results

# Human Behavior During Multi-Attribute Decision-Making with Known Attentional State

The majority of eye movements made were either withinoption or within-attribute saccades (Fig. 1C). Participants generally followed one of two attribute sampling strategies that we call filtering and exhaustive, respectively (Fig. 1D). The filtering sampling strategy began with within-attribute saccades to the participant's preferred attribute of interest, followed by within-option saccades in the option containing the superior magnitude of the attribute of interest. In the  $4 \times 4$  experiment, the majority of participants using this strategy would perform at least two within-attribute sweeps followed by a within-option inspection before their choice. The exhaustive sampling strategy was comprised primarily of sequential within-option saccades to each option. In this strategy, fixation patterns were typically spatial, inspecting options left to right or top to bottom. For additional details of the experimental paradigm and sampling strategy classification criteria, see Methods.

 $<sup>^2</sup>$  Similar results were obtained using the negative log-likelihood and prediction accuracy measures which were used in Table 2.

Fig. 3 Confusion matrices for the  $2 \times 2$  task (top) and the  $4 \times 4$  task (bottom)



Model used to

DFT

aDFT aaDFT

DBS

aDBS

0.0 0.20 0.13 0.0 0.0

0.0 0.33 0.0 0.0 0.0

0.0 0.0 0.0 0.0 0.0

0.0

0.0 0.0

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0.0 0.0 0.0

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0.0

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0.0 0.0 0.0

085

0.0 0.0

2085

0.67

aaDF

2DF

0.0 0.40 0.0 0.0 0.0 0.27 0.0 0.0 0.0 0.0

0.0 0.53 0.0 0.0 0.0 0.0 0.0 0.0 0.13 0.0

Sr.

0.0 0.0 0.07 0.0 0.0 0.0 0.0 0.0 0.0

BIPT

0.0 0.07 0.0 0.0 0.0 0.0 0.13

PROMETHEE SDDW

0.0 0.0 0.07 0.0 0.0 0.27 0.0

BIEN

AMP

Best fitting model

We applied a permutation test to each participant's fixations to determine whether they used either of these strategies. First, we aligned all fixations to the first unique within-option inspection. Next, we determined which optiontype classification best described the first within-option inspection. Option types could be classified by their superior attribute type, second-best attribute type, expected value, variance, or spatial location (left to right or top to bottom). We repeated this procedure for all additional unique withinoption inspections to the remaining options. We compared the overall proportions of the classified option types to a null distribution for each unique within-option inspected. For each participant, the null distributions were determined by shuffling fixations within each trial and recording the largest proportion of the resulting classified option type. This procedure was repeated 1000 times. Filtering classification required each unique within-option inspection to be best classified by the preceding within-attribute fixations (e.g., within-attribute fixations to the probability attribute prior to inspecting the option containing the superior probability magnitude). Exhaustive classification required each unique within-option inspection to be spatially ordered. In total, 13/34 (38%) and 17/21 (81%) participant sampling strategies were classified as filtering in the  $2 \times 2$  and  $4 \times 4$  experiments, respectively. 21/34 (62%) and 4/21 (19%) participant sampling strategies were classified as exhaustive in the  $2 \times 2$  and  $4 \times 4$  experiments, respectively.

Participants significantly preferred the Prob+ option above all others in the 2  $\times$  2 (p < 0.001) and 4  $\times$  4 (p <0.001, paired t-test) experiments (Fig. 4A, B; see Section S2 for details). Thus, most participants were risk-averse and weighed the "probability to win" attribute stronger than any other in estimating the value of an option. The next most important attribute was "amount to win" (amount+ option chosen), while the "amount to lose" (Loss+; only in the  $4 \times 4$ experiment) and "delay to feedback" (Delay+, only in the  $4 \times 4$  experiment) attributes influenced choice much less. These choice preferences were consistent, irrespective of task complexity. When options were ranked by their expected value (EV1 > EV2 > EV3 > EV4), participants significantly preferred the option with the superior EV over all others (p < 0.001, paired t-test) with the exception of EV3 vs. EV4 in the  $4 \times 4$  task (Fig. 4C, D). EV was defined as  $EV = x \cdot p$  in the 2  $\times$  2 task, and

$$EV = \frac{x \cdot p + l \cdot (1 - p)}{1 + k \cdot d} \tag{1}$$

in the  $4 \times 4$  task (see Section S3.1 for further details). We use hyperbolic discounting for the delay variable because it is commonly encountered in human behavior. Last, we ranked options by their risk or variance of outcome VAR.

Variance was defined as  $VAR = x^2p(1-p)$  in the 2 × 2 task, and  $VAR = (x - l)^2p(1 - p)$  in the 4 × 4 task. Options were ranked by their variance (Var1 > Var2 > Var3 > Var4). In the 2 × 2 experiment, participants significantly preferred the option with the lowest variance (p < 0.001, paired t-test). In the 4 × 4 experiment, participants significantly preferred the options with three largest *VAR* values over the Var4 option (p < 0.05, paired t-test), with most participants preferring the intermediate *VAR* options (Var2, Var3). These results imply that participants were not averse to choosing high variance options, but that outcome variance was not heavily weighted during the decision-making process in general.

Task complexity is reflected in the different numbers of fixations before choice and in reaction times, both of which increase substantially from the simple to the complex task (Table 1). An increase in the number of fixations during information sampling from the simple to the complex case is expected because the available information increases from four items (attributes) to 16. This increase is, however, not linear: we find that the number of fixations/item increases from 7.1 fixations/4 items = 1.775 fixations/item to 18.3 fixations/16 items = 1.1 fixations/item. This sublinear relation is consistent with our observation that in the simpler case, nearly all available information was sampled before a choice was made: on average, 94.5% of attributes are fixated at least once before choice in the  $2 \times 2$  task; see Fig. 5A). In contrast, in the more complex case, a substantially smaller fraction of the available information was sampled before choice  $(4 \times 4)$ task: on average, 58% of attributes are fixated at least once before choice). Thus, in the more complex case, a substantial amount of available information was ignored before a choice was made. This is remarkable because no time limit was set for sampling and the participants indeed spent considerably longer time inspecting the choice menu in the more complex task.

We found that, furthermore, the attributes and options that were inspected and, conversely, were ignored are not randomly distributed. The participants most often inspected the "probability to win" attribute (85% across all four options) and least often the "delay to outcome" attribute (30% across all four options). Likewise, the participants most often inspected attributes belonging to the Prob+ option (on average 72%). Attributes belonging to the other three options were inspected less often (Win+: 57%; Loss+: 50%; Delay+: 51%). We surmise that this preference for inspecting certain attributes and options reflects the relative weight of these attributes in determining value and the likelihood that a particular option will be chosen.

The fact that not all information is used to influence a decision and that the pattern of sampled information is correlated with the choice preferences of the participants suggests that **Fig. 4** Choice frequency of each option type averaged over all participants. Win+ (yellow), Prob+ (blue), Loss+ (red), Delay+ (green). **A** 2 × 2 trials, **B** 4 × 4 trials. Option types defined by expected value (EV). **C** 2 × 2 trials, **D** 4 × 4 trials. Option types defined by risk or variance of outcome (Var). **E** 2 × 2 trials, **F** 4 × 4 trials. Significance: \* : p < .05; \*\* : p < .01; \*\*\* : p < .001; < \*\*\* : p < .001, paired t-test



models of decision-making can become more accurate in taking the sequence and content of the sampled information into account. This seems to be more important with increasing complexity of the decision task set, but even in the simple  $2 \times 2$  task set, where all information is sampled, this order could influence the decision process.

#### **Predicting Choice Behavior from Eye Fixations**

Table 1Mean number of fixations and mean reaction times (RTs) fortasks of different complexity. Only data from non-dominated trials areincluded

Experiment	# of fixations	RT [s]
20pt-2att (2x2)	7.1 (SD=1.8)	3.0 (SD=1.0)
4opt-4att (4x4)	18.3 (SD=6.5)	12.1 (SD=3.8)

There is a growing body of work supporting the idea that attentional mechanisms play a causal role in preference development and choice. It was found (Shimojo et al., 2003; Krajbich et al., 2010; Krajbich & Rangel, 2011) that gaze time and frequency correlate with the probability of choosing a given option by eye position as an analog of attention. To examine whether the influence of gaze on choice in our



**Fig. 5** Population mean percentage of information sampled for each option/attribute type. Percentage represents the overall proportion of trials where an attribute type (rows) within an option type (columns) was sampled. A  $2 \times 2$  trials, **B**  $4 \times 4$  trials. A Kolmogorov-Smirnov test was used to determine if the distribution of population information

sampling frequencies for a *single* attribute type within an option type was significantly greater than the distribution of population information sampling frequencies for *all* attribute types within *all* option types. \*: p < .05; \*\* : p < .01; \*\* \*\* : p < .001; \*\* \*\* : p < .0001, Kolmogorov-Smirnov test

experimental paradigm was consistent with previous studies, we performed a logistic regression to predict choice from the difference in expected utilities (EUs) of two options (Thomas et al., 2019) coded as 0 (unchosen) or 1 (chosen). For  $4 \times 4$ trials, the left- or top-most option was coded as 0, and the remaining option with the greatest EV was coded as 1. Using logistic regression, we estimated the probability of choosing the left- or top-most option given the relative option value (the difference between the left- or top-most option and the remaining option with the highest EV). Regression parameters were fit to individual subjects based on their choice behavior. For each trial, the difference in residuals between actual choice probability and those predicted by the difference in EU were averaged separately for trials with positive or negative gaze advantage. Gaze advantage was computed as the difference in the proportion of the number of fixations made to one option compared to all others (Fig. 6, top), the difference in the proportion of the total gaze duration on one option compared to the time spent fixating on all others (Fig. 6, middle), and their difference (Fig. 6, bottom). Finally, we computed the average difference in choice probability for options with positive versus negative gaze advantages when adjusted for EU influence. Figure 6 shows the gaze advantage in predicting choice behavior. At the group level in the  $2 \times 2$ 

**Fig. 6** Gaze influence on choice. Mean increase in choice probability for an option due to increased fixation count (upper panel; blue), longer total gaze duration (middle panel; red), and their difference (lower panel; grey) after accounting for the influence of option value





experiment, the advantage has a mean value of 0.23, with a standard deviation (SD) of 0.16 for the number of fixations, 0.26 (SD=0.18) for the gaze duration, and < 0.01 (SD=0.04) for their difference. At the group level in the 4 × 4 experiment, the mean advantage has a value of 0.07 (SD=0.06) for the number of fixations, 0.06 (SD=0.05) for the gaze duration, and 0.02 (SD=0.03) for their difference.

In addition, to confirm the influence of gaze on choice, we applied two versions of an expected utility × gaze regression model (Glickman et al., 2019), in which the EU of each option increases with the amount of time it is fixated (Fig. 7A) or the number of times it is fixated upon (Fig. 7B). The multiplicative value of each option is computed as  $x^{\alpha} \cdot p$  (2 × 2 experiment) and  $\frac{x^{\alpha} \cdot p + l^{\alpha} \cdot (1-p)}{1+k \cdot d}$  (4 × 4 experiment), see Eq. 1 and text immediately preceding it. The variables *x*, *p*, *l*, and *d* 

are option amount, probability, loss, and delay, respectively. The following parameters are fit for each participant separately. *k* is a discounting parameter fit to each participant that describes how steeply delay diminishes value.  $\alpha$  is the individualized risk parameter of EU (how likely each participant would choose an option with a lower probability of winning a higher amount compared to an option with a higher probability of winning a lower amount). In Fig. 7A and B,  $\tau$  is a saturation parameter that reflects the factor by which dwell time or fixation number increases the value of the fixated option.  $\beta$  is a slope parameter indicating the sensitivity of the model to the difference in *EU*. Both dwell time and fixation count models displayed similar AIC values (Fig. 7C, E) and improvements in prediction accuracy (the proportion of correct choices predicted by the model) from the traditional



**Fig. 7** Expected utility-based regression models. **A** Dwell time *EU* model: the *EU* value of each option is influenced by the proportion of time spent looking at each option. **B** Fixations *EU* model: the *EU* value of each option is influenced by the proportion of fixations made to each option.  $2 \times 2$  trials: **C** AIC for the traditional, dwell time and number

of fixations EU. **D** Prediction accuracy for the traditional, dwell time and number of fixations EU.  $4 \times 4$  trials: **E** AIC for the traditional, dwell time and number of fixations EU. **F** Prediction accuracy for the traditional, dwell time and number of fixations EU

*EU* models (Fig. 7D, F) where eye movement information is not considered. Overall, this improvement in prediction accuracy was more pronounced in the  $4 \times 4$  condition relative to the  $2 \times 2$  condition.

# Choice Prediction Performance of Computational Models

The free parameters in all models introduced in "Computational Methods" section were fit to the data based on the training set, and performance of the models was determined on the test set. Fitted parameter values and their distributions for all 15 models, both tasks, and all subjects are shown in Supplemental Section S7 . We performed a model recovery study, see "Model Recovery and Parameter Recovery" section and Supplementary Section S9, to show that this approach can distinguish the models. The resulting confusion matrices show that this is the case for some but not all of the models.

Table 2 summarizes for all models their performance in predicting the choices participants made in the simple task (2 options, 2 attributes) and the complex task (4 options, 4 attributes). For each task, we rank the models according to how accurately they explain the choices on the test set (last column), i.e., the subset of observed choices that was *not* used to optimize model parameters. The following sections describe the factors that contribute to differences between the performance of different models.

# Model Ranking and Determination of the Best-Performing Models

As mentioned, the models in Table 2 are ranked by increasing cross-validated prediction accuracy on the test set. To define the best model, the highest-ranked model is compared with the second-ranked model, then third-ranked model, etc., until a significant difference is found. Models without significant difference are all considered as the best models. In the  $2 \times 2$ task, there is one clear "winner," i.e., one model whose performance is significantly better than all other models: The LCA-PT has significantly greater prediction accuracy on the test set (p = 0.00011, paired t-test) compared with aDDM, which is ranked second. In addition, it exceeds the performance of all other models in all four criteria: it has the lowest AIC score and lowest negative log-likelihood and the highest prediction performance on both the training and the test set. This consistency increases the confidence that the model is superior over the other tested models. Our result is in agreement with a prior finding (Glickman et al., 2019) where the performance of the LCA-PT model was highest in a related decision-making task of comparable complexity. It predicted the choice behavior of a non-overlapping set of observers better than all other tested models.

In the 4 × 4 experiment, the AMP has highest prediction accuracy on the test set. Its performance is not statistically different from that of the second-ranked model which is LCA-PT (p = 0.30, paired t-test), and performance of these two models is significantly better than that of the thirdranked model (AMP to PROMETHEE, p = 0.025; LCA-PT to PROMETHEE, p = 0.013; paired t-test). In absolute numbers, the AMP model received the best scores in all four measures. We also found that performance differences between models are much starker in the 4 × 4 experiment than in the 2 × 2 experiment. In particular, in terms of prediction accuracy, both test and training, the AMP model's performance was more than twice as high than that of the lowest-performing model (DbS).

# Varying Task Complexity Influences Model Performance

We study the performance of 15 different models for predicting choices of human participants in two sets of gambles of different complexity. For the set consisting of simple gambles (2 options with 2 attributes each), we find a relatively narrow range for the prediction accuracy. On the training set, prediction accuracy varies from a low of 81.4% (for exhaustive DbS) to a high of 92.4% (LCA-PT). The range is even slightly smaller, 81.6% (exhaustive DbS) to 92.0% (LCA-PT) for the test set, see Table 2. Though this reflects some variation in model performance, all of these very different models seem to be able to predict behavior reasonably well, all making correct predictions for better than 80%, and sometimes 90%, of choices. We are possibly looking at a ceiling effect that does not allow us to meaningfully differentiate between the tested models. We note that this is not the case for either the AIC measure nor the negative log-likelihood: in both measures, the spread between models is much larger. However, the two lowest-performing models for prediction accuracy also have the worst measures for AIC and negative log-likelihood, and the best-performing model for prediction accuracy has the best results for these measures.

For the complex gambles (4 options with 4 attributes each), the first observation is that overall prediction accuracy is generally lower (this is not the case, however, if prediction accuracy is compared to chance which is 50% for the  $2 \times 2$  task and 25% for the  $4 \times 4$  task). Each model performs less well in this situation than the equivalent model in the simple situation. Overall lower prediction accuracy for a more complex situation is, of course, not unexpected. More interesting is that for the more complex choices, the *differences* between models vary considerably more than in the simpler task. Prediction accuracy on the test set varies from a low of 38.1% (exhaustive DbS) to a high of 87.0% (AMP), i.e., by more than a factor of two (again, chance level is 25%). Results are similar for the training set. To make sure that

this is not simply due to a larger amount of data collected in simple gambles (540 trials per participant in 2 × 2 case versus 150 in 4 × 4 case), we reran the analysis on the former using only the first 150 trials of each participant. No significant effect on prediction accuracy was found for the number of trials (two-way ANOVA,  $F_{1,990} = 0.117$ , p = 0.733), nor was there a significant interaction effect between the number of trials and choice of model (two-way ANOVA,  $F_{14,990} = 0.730$ , p = 0.745). However, as expected, the choice of models contributes significantly to the accuracy (two-way ANOVA,  $F_{14,990} = 7.69$ ,  $p = 1.34 \times 10^{-15}$ ). It thus appears that, even though there is one model that significantly outperforms all others in the commonly used 2-option, 2-attribute choice task (LCA-PT), overall, this task design may not be very suitable to differentiate between models, at least if choice prediction is chosen as the criterion.

The pronounced differences of choice prediction performance for the complex 4-option, 4-attribute task allow us to differentiate between the model classes more clearly. As mentioned, one model (AMP) outperforms all others, according to all four criteria in Table 2. However, the difference between the AMP model and the next best-performing model

**Table 2** Summary of results for the  $2 \times 2$  (top) and  $4 \times 4$  (bottom) tasks

		# of params	AIC (training)	Prediction accuracy (training)	-2 Neg. log-likelihood (test)	Prediction accuracy (test)
2×2	DbS	2	10589	81.4%	2671	81.6%
	EV	1	10678	84.1%	2692	84.1%
	aDbS	1	9682	85.5%	2454	85.3%
	DFT	10	9544	85.8%	2756	85.4%
	aDFT	7	10445	86.1%	2710	86.0%
	BI-EV	1	10027	86.1%	2590	86.1%
	PROMETHEE	5	8627	87.5%	2212	87.1%
	AR	3	8986	87.6%	2297	87.4%
	BI-PT	3	9311	87.5%	2425	87.5%
	AMP	5	8961	88.3%	2271	87.8%
	aaDFT	7	8483	88.7%	2138	88.4%
	PT	3	8499	89.6%	2230	89.0%
	LatentVariable	6	8821	89.5%	2083	90.4%
	aDDM	6	7258	91.1%	2003	90.9%
	LCA-PT	5	6549	92.4%	1674	92.0%
4×4	DbS	2	4953	38.0%	1338	38.1%
	EV	2	2753	73.4%	704	73.1%
	BI-EV	2	3338	76.9%	865	76.8%
	DFT	14	2607	77.6%	791	77.4%
	aDbS	1	2532	78.1%	650	77.9%
	BI-PT	6	3188	79.6%	806	79.4%
	aDFT	9	2647	80.1%	636	79.5%
	aaDFT	9	2580	81.1%	612	80.7%
	aDDM	9	2666	81.7%	874	81.6%
	LatentVariable	12	2288	85.3%	582	81.7%
	PT	6	2097	84.5%	633	81.9%
	AR	5	2130	83.8%	576	82.2%
	PROMETHEE	7	2177	84.0%	618	82.2%
	LCA-PT	8	1940	87.1%	480	85.9%
	AMP	5	1755	87.8%	471	87.0%

Shown are number of free parameters used for fitting per subject, Akaike information criterion (AIC), choice prediction accuracy for the training set, negative log-likelihood, and choice prediction accuracy for the test set. All results were subject to five-fold cross validation (Section S3.5). Models are ranked based on their prediction accuracy on the test set (last column). Within each task, bold numbers indicate that prediction accuracy for that entry is significantly better than for all entries with non-bold numbers

(LCA-PT) is not significantly different in any of the four criteria, making the two models statistically tied in their performance. We note, however, that AIC differences exceeding 10 are considered very strong evidence in favor of the model with the lower numerical values (Fabozzi et al., 2014). By this criterion, the performance of the AMP model then exceeds that of all others, including LCA-PT.

# Sequence of Attentional Selections Strongly Affects Decisions

One of the main questions we want to address in this project is whether the detailed sequence of eye movements, i.e., attentional deployment, does influence human behavior. Alternatively, eye movements could be a random process for gathering information where the choice is made based on that information, but the order in which any particular piece of information is acquired does not matter. To answer this question, a comparison between the two decision by sampling (DbS) models is useful because they are identical, except that one takes into account the specific eye track and the other does not. We find (Table 2) that the DbS model with attentional influence performs far better than its exhaustive version where no attention history data of participants is used. The former model's accuracy on the test dataset exceeds the latter by 3.7 percentage points in the  $2 \times 2$  experiment and by 39.8 percentage points in the  $4 \times 4$  experiment. It is clear that the attentional history contains crucial information for predicting the participants' decisions, and this is especially important when the task is more complex. That sampling of attributes and objects is not uniform, or stochastically with a uniform distribution, is reinforced by our behavioral findings which show that only 58% of the available information is used before the choice is made in the  $4 \times 4$  case.

# Asymmetry of the Effect of Positive vs. Negative Attributes on the Choice

In the AMP model, the first stage of the decision process is based on pairwise comparisons between attributes, i.e., the attended attribute of the attended option over the corresponding attributes of the non-attended option(s). The result of this comparison can either be positive (advantageous) for the attended attribute) or negative (disadvantageous). This is quantified in the advantage function, Eq. 39. The parameter  $\rho$  determines the weight of a positive vs. negative difference between attributes. Differences are weighted equally for  $\rho = 0.5$ , with  $\rho > 0.5$  indicating that positive information is weighted higher towards the decision, and  $\rho < 0.5$ the opposite. We find on average  $\rho = 0.814$  in the 2 × 2 experiment and a significantly higher  $\rho = 0.961$  in the 4 × 4 experiment (p = 0.0013, Welch's t-test). We note, however, that  $\rho$  was only recoverable in the 2 × 2 experiment, not in the  $4 \times 4$  experiment, so the statement in the previous sentence needs to be treated with caution. The same applies to the next paragraph.

At least in the simpler task, the value of attended attributes therefore contributes more to the choice of an option when it is favorable compared to when it is non-favorable. In other words, participants weigh positive/advantageous information higher than negative/disadvantageous information. This "optimistic filtering" strategy may be explained by the ultimate goal of the decision-making task which is to choose the best option rather than rejecting inferior options (Sepulveda et al., 2020; Glickman et al., 2018). Systematic application of a rejection strategy would require the knowledge of all options, which can be expensive under many scenarios. Thus, focusing mainly on positive information may be needed to decrease the cognitive load by lowering the number of items that need to be attended and/or kept in memory.

# Working Memory of Integrated Attributes Retained Between Fixations

In this section, we consider memory losses in the AMP and LCA-PT models at the processing level at which attributes have been observed and, at least potentially, integrated across attribute values. For the LCA-PT model, the factor  $(1 - \psi)$ is a measure of the decay of attribute memory from one fixation to the next, Eqs. 27-30. Its mean value is 0.266 in the  $2 \times 2$  experiment and 0.524 in the  $4 \times 4$  experiment. For the AMP model,  $\delta$  controls the proportion of the advantage matrix that is carried over to the next fixation, Eq. 31. Its mean value is similar in both experiments, 0.707 in the  $2 \times 2$ experiment and 0.784 in the  $4 \times 4$  experiment. In all these cases, a considerable proportion of information is therefore lost between fixations. Even for the AMP model where the decay is weaker, and for the smallest number of fixations until choice (on average, 5.6 fixations for the  $2 \times 2$  model, see Table 1), the mean retained information from the first fixation to the last is less than  $15\% (0.707^{5.6})$ . The retained information is negligible ( $< 10^{-3}$ ) for all other cases.

One possible interpretation is that the decision is highly dependent on the last several fixations only. But what is then the role played by fixations early on in each trial? Is the information collected in these fixations essentially discarded without playing a role in the decision process? While we cannot exclude this possibility given the data presented, there are at least two other interpretations.

The first explains this finding as a lack of correlation between observations in early fixations and eventual option selection. During the first fixations, participants have no information about the value of many attributes because they have never seen them. Only after a minimum of four (in the  $2 \times 2$  task) or 16 (4 × 4 task) fixations have been executed can all values potentially be known. A substantial fraction of participants may therefore sample the display components either in random order (because they assume the order does not matter) or in an idiosyncratic, stereotypical order (because it is easiest to follow the same order in each fixation, and an order going mostly between neighboring symbols or symbols corresponding to the same attributes or options may appear natural). The present study does not address this question, but we, indeed, found evidence for stereotypical behavior for the first fixations when we performed formal analyses of fixation orders (Elsey et al., in preparation). Data obtained in these early fixations then serve to guide which attributes are sampled later. However, the attribute values observed early in the search, during either idiosyncratic sequences or random sequences, are not correlated with attribute values observed later, close to the decision. This lack of correlation results in low values for  $(1 - \psi)$  in Eqs. 27–30 and for  $\delta$  in Eq. 41.

A corollary of this hypothesis is that values for  $(1-\psi)$  and  $\delta$  should be considerably higher for "re-visiting" fixations, i.e., fixations landing on attributes that had been sampled at least once before. When we ran this analysis, to our surprise, we found the opposite: Both of these parameters, for both tasks, were lower than when all fixations were included. In the 2 × 2 task, we now found  $(1 - \psi) = 0.08$  (LCA-PT),  $\delta = 0.39$  (AMP) and in the 4 × 4 task,  $(1 - \psi) = 0.38$  (LCA-PT),  $\delta = 0.64$  (AMP). Furthermore, the prediction accuracy of both models, again for both tasks, was lower than when all fixations were included. In other words, including the first fixations improved performance, knowledge of the values of the first attributes influences the choice.

Taken together, our data are in agreement with a process in which input from both the first fixations as well as the last fixations contribute substantially to the choice, more than those at intermediate positions in the sequence, We thus observe both a primacy effect (high impact of first fixations) and a recency effect (high impact of last fixations). This is a wellknown phenomenon of memory recall which is commonly called the serial position effect (Deese & Kaufman, 1957). Obviously, our assumption that memory contents decays exponentially does not capture this effect. We expect that better model performance can be achieve by replacing the simple monotonic decay of memory contents by a more realistic model. However, this goes beyond the scope of the current study and is reserved for future work.

#### **Inter-trial Effects Are Weak**

What is the influence of the winning/losing history in previous trials on the choice in the current trial? Even though in our experiment trial outcomes are independent, humans are subject to biases like the hot-hand fallacy or gambler's fallacy (Rabin & Vayanos, 2010; Sacré et al., 2019) which may create interactions between trials. While such interactions cannot improve the participants' gain (sum of rewards), taking them into account in modeling their responses can improve prediction of their responses. We therefore developed in Section S3.2.1 the latent variable model, a variation of prospect theory in which the reward history modifies model parameters by taking into account dependencies between trial outcomes.

We find that the additional degrees of freedom in the latent variable model improve prediction accuracy on the test set slightly in the  $2 \times 2$  experiment (by 1.4 percentage points, the difference is significant, p = 0.011, paired t-test) over that of standard prospect theory, see Table 2, as well as in terms of negative log-likelihood. However, the opposite is the case for AIC and for the prediction accuracy on the training set. For the more complex  $4 \times 4$  task, the latent variable model is slightly better than prospect theory as far as prediction accuracy on the test set and on the training set is concerned (by 0.2 and 0.3 percentage points, respectively) but worse for the AIC and negative log-likelihood measures. Note that since the latent variable model is a generalization of prospect theory, a perfect optimization procedure should reduce the former to the latter if the addition of between-trials interactions reduces the model's predictive performance. However, this would require that all ten free parameters in Eq. 10 are set exactly to their required values (all  $r_x = 0$  and all  $b_x = x$ , for  $x \in \{\alpha, \beta, \gamma, \lambda, k\}$ ). In this case, the latent variable LV can still formally be computed, but it has no effect and the parameter a becomes irrelevant. We surmise that our minimization procedure is not capable of reaching this global minimum in the high-dimensional landscape of the optimization problem, and that for this reason, prediction accuracy on the test set (the cost function for the optimization) is worse despite the availability of the additional (latent) variables.

Overall, it is fair to say that the impact of the latent variables is small and that taking into account interactions between trials brings little or no improvement to prospect theory.

#### Discussion

#### **Measures of Complexity**

We are confident that our experimental design using four options with four attributes each is more "complex" by any reasonable measure of complexity, but we do not attempt to formally quantify the "degree" of increase in complexity. This is mainly because in the complex task, we add two attributes (amount of potential loss, delay to reveal of outcome) to the two attributes in the simple design (amount to win, probability to win it). Taking into account two additional factors by itself increases the complexity of the task, but there are other factors that likely contribute too. For instance, in the simple task, the outcome is either winning the specified amount or not; there is never a loss. In contrast, the complex gamble involves both a potential win and a potential loss. Combining these two factors, vs. only one factor in the simple task, requires additional mental processing, a hallmark of complexity of cognitive operations (Oprea, in press). Likewise, taking into account the delay variable increases additional processing which may include temporal discounting but also hedonic cost, e.g., in the form of effort or mental suffering. In this study, we do not attempt to quantify these differences (although the subjective increase in hedonic cost was very obvious in pilot experiments). Oprea (in press) attempts to operationalize the concept of complexity in human information processing, and such approaches may allow to use more rigorous approaches in future work.

We also note that our main interest is in studying the effect of task complexity on choice selection, and we use a stochastic (risky) paradigm mainly because it is a convenient and widely used experimental design. While it is not always possible to extend conclusions based on one experimental paradigm to others, recent work shows that some mechanisms that previously have been considered specific to risky choices can be explained, at least to a large extent, to varying levels of task complexity (Oprea, 2024).

#### **Highest Performance Is Achieved by Process Models**

One conclusion we can draw from the comparison between the computational models is that, overall, process models tend to perform better than static models. The three bestperforming models in both the  $2 \times 2$  and  $4 \times 4$  experiments are process models, see Table 2. It would be wrong, however, to conclude that all process models are superior to all static models. Indeed, the fourth-best model in both experiments is a static model (prospect theory in the  $2 \times 2$  and AR in the  $4 \times 4$  experiment, respectively). Furthermore, not all process models are among the high performers. Among the lowest performers in both experiments are two process models, the aDbS and the BI-EV model.

# Basic Assumptions of Prospect Theory and of the AMP Model

The static models used in our model comparison are the classic economic models of risky choice. In particular, prospect theory is the currently dominant model in behavioral economics (Ruggeri et al., 2020). While not coming out at the top in any category, prospect theory is among the best performers on our behavioral data set. This warrants a comparison of its defining features with those of the AMP model, the best-performing model in the complex decision situation. An important difference is that the latter does not make any of the basic assumptions of prospect theory, like the specific forms for the computation of expected utility or the nonlinear form of the influence of probability to win. Of course, it makes other assumptions, like the specific working memory model we use or the computation of relative advantages. These concepts are, however, closer to being interpretable in terms of neuronal processing than the purely functional constructs of prospect theory.

We also point out that computation of expected value or utility, of any form, is nowhere required in the AMP model. It has been argued many times (Stewart et al., 2006) that this is a difficult quantity to assign to individual options, while, on the other hand, relative value *differences*, which are fundamental to the PROMETHEE and AMP models, are much easier to determine. For the simple task, it may be possible for at least a sizable fraction of participants to compute some approximation of an explicit option value (EV, utility, etc.) resulting in good or even excellent results for models based on this computation, like leaky accumulators or prospect theory. This may not be possible any more for the more complex case where attribute differences remain easier to compute, favoring the AMP model.

# Relation of the AMP Model to the Context-Dependent Preference Model

Some of the intuitions in the AMP model are similar to those in a context-dependent model by Tversky and Simonson (1993). That model combines the effect of background context (options encountered in the past) and local context (offered option set in current trial), with the background affecting the global change in the relative weight of the attributes. In our model, this is captured by trial-specific attribute weights  $\omega_{\pi}(n)$  (the relative number of fixations to attribute  $\pi$  in all trials up to the current one, trial *n*; this is defined after Eq. 40). Attention to an attribute is taken as an indicator of the importance a participant assigns to it, compared to other attributes.

A second assumption in the Tversky and Simonson model is that the effect of local context can be interpreted as a "tournament" in which the candidate option is matched against all the other presented options, and its overall score is the sum of the results of these matches. This is also the case in our model where several pairwise comparisons occur at every fixation and the final decision is made from the accumulation process of advantage values. Tversky and Simonson assume that the disadvantage of option i over option j should have at least the same impact as the advantage of option j over option i. In contrast, in our model, we are agnostic to the relative impacts of advantages and disadvantages of options, and we allow them to vary unconstrained between participants.

The relative impact is controlled by the parameter  $\rho$  in Eq. 39. It sets the influence of the contribution of the advantage vs that of the disadvantage of an attribute directly. In the  $2 \times 2$  task, we find that for a very large majority of partici-

pants, this parameter takes essentially the value of unity, see Fig. 22. Thus, the net impact of advantages strongly dominates that of its disadvantages. The situation is similar in the  $4 \times 4$  task; however,  $\rho$  was found to be not recoverable, so any interpretation based on its value distribution has be to treated with caution. Also, this effect appears to be the opposite of the commonly found observation of loss aversion. It is important to note, however, that loss aversion is usually, including by Tversky and Simonson (1993), formulated when comparing *options*, while  $\rho$  concerns the comparison of *attributes*.

# **Limitations and Future Work**

Behavioral data were collected from a total of 50 human volunteers (number determined by power analysis, see Methods) that performed in our eye tracking experiment. One limitation of the experimental design is that the spatial configuration differs between the simple and the complex task. This may encourage within-attribute processing in the complex task, because the attributes are aligned (and easy to scan). We believe this is a minor factor because another study with the same spatial configuration of the complex task in this study but the same number of options and attributes in the simple task agreed with our findings in the simple task (Glickman et al., 2019).

In the 2  $\times$  2 task, the location of the symbols containing the attribute information (blue and yellow masks and symbols indicating quantities below) are assigned random positions within the option symbols (gray bars). In contrast, in the 4  $\times$  4 task, these symbols are at fixed locations, except for global flips between all option symbols oriented either vertically or horizontally. Future work should study how randomizing attribute locations in the 4  $\times$  4 task affects behavior.

The two factors we used to increase the complexity from the simply to the complex task are delay and amount of potential loss. Both are known aversive components in primate decision-making. It would be of interest, however, to study to what extent our results generalize to other factors.

We evaluate a number of computational models from a variety of "families." Some are classical models that have been in use literally for centuries (EV), while others were specifically designed by us for this study (AMP, exhaustive DbS, DbS with attention, aDFT, aaDFT). Models range in complexity from simple models with one free parameter to the most complex with 14. Models come from a variety of fields, mainly from economic theory but also cognitive science/psychology, operations research/management science, neuroeconomics, etc. Despite this diversity of models along several dimensions, there are commonalities between all of them. It is likely that other models differ in these respects and that this limits the range of phenomena that the models we study can explain. It would be very difficult, and go way

beyond the scope of this study, to include in the simulations all possibly applicable models.

A limitation that concerns all process models we consider (described in Sections S3.2 and S3.3) is that attention has a sequential effect on action value updating. This framework is consistent with a large body of work that sees selective attention as a mechanism designed to deal with large amounts of information that needs to be handled, in excess of what can be processed in detail by the brain (Broadbent, 1958). Selective attention solves this problem by identifying the instantaneously most relevant information and suppressing processing of all other input. The relevant information is then processed sequentially in some priority order (Itti et al., 1998; Niebur & Koch, 1998). It has also been proposed that attention can be divided between targets, either by genuinely parallel processes or by rapid shifting of the focus of attention (Corbetta et al., 1991; Johnson & Zatorre, 2006). Theoretical analyses have shown that a distinction between serial and parallel processes is not trivial, e.g., by Townsend (1972, 1990). While our models are compatible with "semi-parallel" processing emulated by fast switching between sequential stages, truly parallel mechanisms likely require model architectures that none of the 15 models considered in this study covers. Furthermore, most of the cited work was done to understand mechanisms of perceptual attention, but the basic principles may also apply to the use of attention in decisionmaking.

Another limitation of our modeling work is that we do not make use of all data that we gather in the empirical part of this study. Our strategy is to use fixation data generated in response to the stimuli presented and use models to explain the choices the participants make. We do, however, also have access to the times when these choices are made, i.e., the reaction times (RTs). Two recent papers (Evans et al., 2019; Molloy et al., 2019) have shown that including RT distributions can play an important role in constraining computational models used to understand context effects in multi-attribute decision-making, in particular violations of normative theories like the attraction, similar and compromise effects. Molloy et al. (2019) showed that estimates of parameters of the influential multi-attribute linear ballistic accumulator (MLBA) model are improved by adding RT data to the commonly used (including by us) choice data alone. Evans et al. (2019) add three more computational models to the MBLA, including two that are closely related to some that we include in our study (LCA and multi-alternative decision field theory, MDFT). They argue that using only choice behavior can lead to spurious conclusion in the study of the mentioned and potentially other context effects.

It would be of great interest (and it is a target for future work) to compare our approach with one that uses not only the choice data but also the reaction time distributions. Such a study would be even more useful if it considered separately violations of normative theories like those listed above and more generic decision-making situations like those addressed in our current study. This might answer the question whether RT data is critical to distinguish between computational model performance only in specific contexts, as in the carefully controlled violations of normative theories, or more generally in a large number of decision-making situations.

As a first approach, we have regressed the decision time (sum of all fixation durations in a given trial) over the difference in expected value (with hyperbolic discounting for the 4 × 4 task, as described) between the two options. Averaged over all participants, we find highly significant negative correlations in both the 2 × 2 and the 4 × 4 tasks (correlation coefficients -0.194,  $p = 1.91 \times 10^{-155}$  and -0.115,  $p = 2.08 \times 10^{-9}$ , respectively; Wald's test, null hypothesis: correlation=0). This is consistent with the expectation that more difficult choices (similar expectation values for both options) result in longer decision times. These results may also provide support for the idea that people integrate evidence (which is sensitive to the EV-difference) to a response criterion. Models compatible with this idea (like LCA-PT and aDDM) may be good candidates for future work.

A more general limitation applying to all of our models is that their final choice stages assume a parallel representation of the action value of all available options, which compete with each other. While this mechanism seems to make functional sense, potential use of other selection types is at least conceivable. One example is a random selection from a class of candidate choices. None of the models discussed in this study implements such a choice mechanism.

# Conclusion

We carried out two behavioral experiments in which human participants made risky decisions between lottery options. We varied the level of complexity, and we monitored the attentional state of participants via eye tracking. Fifteen choice models were examined in their ability to predict the choices made by the participants. All models predict choices significantly better than chance. As expected, overall predictive performance is higher for the simpler than for the more complex task. We found that the increase in task complexity leads to a switch from within-alternative to within-attribute processing. Furthermore, there are only small differences in predictive performance between models in the case of the simple task, a possible ceiling effect, while for the complex task, different models are clearly distinguished by the quality of choice predictions. This raises doubts to what extent the simple task, which is commonly used in studies of risky decision-making, is suitable for rigorous comparisons of computational models. Finally, we find that the best-performing models have in common that they take into account attentional behavioral data and that they incorporate explicit memory mechanisms.

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**Data Availability** All data and code (Python and Matlab) used to generate results in this report are freely available through the Johns Hopkins University Research Data Repository at https://doi.org/10.7281/T1/7IXSOP

#### Declarations

**Ethical Approval** All experimental procedures were approved by the Homewood Institutional Review Board of Johns Hopkins University, approval HIRB00009896.

**Consent to Participate** Written informed consent was obtained from all participants.

Competing Interests The authors declare no competing interests.

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# **Authors and Affiliations**

Xinhao Fan<sup>1,2</sup> · Jacob Elsey<sup>1</sup> · Aurelien Wyngaard<sup>1,3</sup> · Aaron L. Sampson<sup>1</sup> · You-Ping Yang<sup>1,8</sup> · Erik E. Emeric<sup>1</sup> · Moshe Glickman<sup>5,6</sup> · Marius Usher<sup>4</sup> · Dino Levy<sup>7</sup> · Veit Stuphorn<sup>1</sup> · Ernst Niebur<sup>1</sup>

- Ernst Niebur niebur@jhu.edu
- <sup>1</sup> Zanvyl Krieger Mind/Brain Institute, Johns Hopkins University, Baltimore, MD, USA
- <sup>2</sup> Institute of Physics, Nankai University, Tianjin, People's Republic of China
- <sup>3</sup> Institut de Biologie, École Normale Supérieure, Paris, France
- <sup>4</sup> School of Psychological Sciences, Tel Aviv University, Tel Aviv, Israel

- <sup>5</sup> Affective Brain Lab, Department of Experimental Psychology, University College London, London, UK
- <sup>6</sup> Max Planck UCL Centre for Computational Psychiatry and Ageing Research, University College London, London, UK
- <sup>7</sup> Cooler School of Management and Sagol School of Neuroscience, Tel Aviv University, Tel Aviv, Israel
- <sup>8</sup> School of Medicine, National Defense Medical Center, Taipei, Taiwan